TI-Nspire™ CAS Assisted Solutions

VCE Mathematical Methods Written Sample Examination 2 Section B

Explanatory notes:

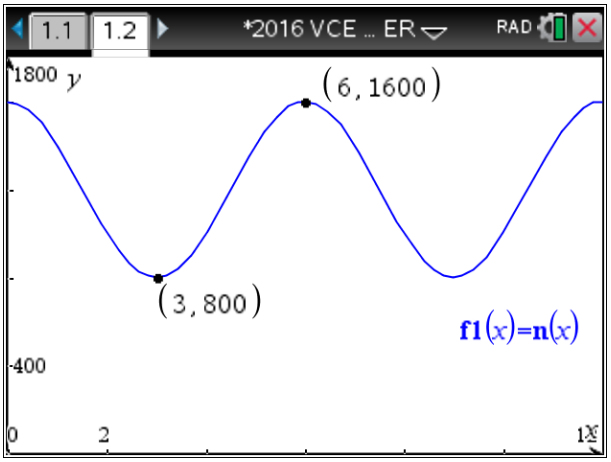
Note that the VCAA only supplies multiple-choice answers to sample papers. Every effort has been made to ensure that these solutions are correct.

The author of these solutions has no affiliation with the VCAA.

SECTION B – Extended response questions

Question 1:

Part (a)

Working	**TI-Nspire CAS screenshot(s)**
$T = \dfrac{2\pi}{\frac{\pi}{3}}$ $= 6$ (months) $A = 400$	

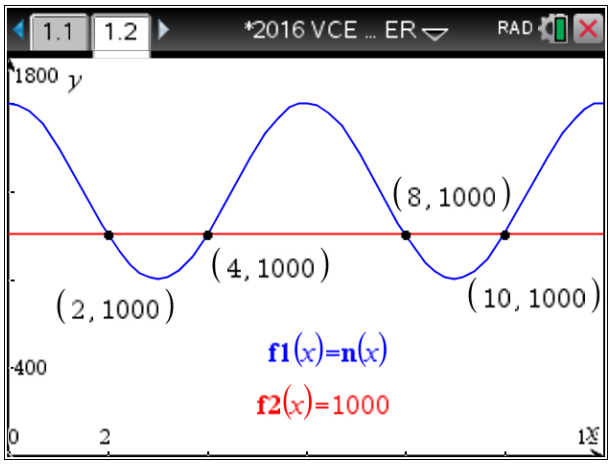
Part (b)

Working	**TI-Nspire CAS screenshot(s)**
From the graph above right, $n_{max} = 1600$ and $n_{min} = 800$.	

Part (c)

Working	**TI-Nspire CAS screenshot(s)**
$n(10) = 1000$	$n(t) := 1200 + 400 \cdot \cos\left(\frac{\pi \cdot t}{3}\right)$ *Done* $n(10)$ 1000

Part (d)

Working	**TI-Nspire CAS screenshot(s)**
$n < 1000$ for 4 months so the fraction of time is $\frac{1}{3}$.	

Question 2:

Part (a)

Working	**TI-Nspire CAS screenshot(s)**
$6480 = 5x^2 + 7hx$ $7hx = 6480 - 5x^2$ $h = \frac{6480 - 5x^2}{7x}$	$6480=2\cdot x\cdot h+2\cdot h\cdot \frac{5\cdot x}{2}+2\cdot x\cdot \frac{5\cdot x}{2}$ $6480=5\cdot x^2+7\cdot h\cdot x$ $6480-5\cdot x^2=5\cdot x^2+7\cdot x\cdot h-5\cdot x^2$ $6480-5\cdot x^2=7\cdot h\cdot x$ $\text{comDenom}\left(\frac{6480-5\cdot x^2}{7\cdot x}=\frac{7\cdot h\cdot x}{7\cdot x}\right)$ $\frac{6480-5\cdot x^2}{7\cdot x}=h$

Part (b)

Working	**TI-Nspire CAS screenshot(s)**
Solving $V(x)>0$ for x with $x>0$ gives $0<x<36$. The Notes application can be used to solve max/min problems.	Max/min problems $\mathbf{v}(x):=\frac{5\cdot x\cdot(6480-5\cdot x^2)}{14}$ ▸ *Done* $\text{solve}(\mathbf{v}(x)>0,x)\mid x>0$ ▸ $0<x<36$

Part (c)

Working	**TI-Nspire CAS screenshot(s)**
$\frac{dV}{dx}=-\frac{75}{14}x^2+\frac{16200}{7}$ where $a=-\frac{75}{14}$ and $b=\frac{16200}{7}$ Author comment: I am uneasy about what working is required for this 3-mark question part when TI-Nspire CAS gives $\frac{dV}{dx}$ directly in its required form.	$\frac{d}{dx}(\mathbf{v}(x))$ ▸ $\frac{16200}{7}-\frac{75\cdot x^2}{14}$

Part (d)

Working	**TI-Nspire CAS screenshot(s)**
Solving $\frac{dV}{dx}=0$ for x with $x>0$ gives $x=12\sqrt{3}$ (cm). Substituting $x=12\sqrt{3}$ into h gives $h=\frac{120\sqrt{3}}{7}$ (cm).	$\frac{d}{dx}(\mathbf{v}(x))$ ▸ $\frac{16200}{7}-\frac{75\cdot x^2}{14}\mid$ $\mathbf{xc}:=\text{zeros}\left(\frac{d}{dx}(\mathbf{v}(x)),x\right)\mid x>0$ ▸ $\{12\cdot\sqrt{3}\}$ $h=\frac{6480-5\cdot x^2}{7\cdot x}\mid x=xc$ $\quad h=\left\{\frac{120\cdot\sqrt{3}}{7}\right\}$

Question 3:

Part (a) (i)

Working	**TI-Nspire CAS screenshot(s)**
$X \sim \text{Bi}\left(20, \frac{5}{8}\right)$ $\Pr(X \geq 10) = 0.9153$ (correct to 4 dp) The Notes application can be used to solve questions involving the binomial distribution.	Binomial Probability Calculations $n := 20 \rightarrow 20$ $p := \frac{5}{8} \rightarrow \frac{5}{8}$ $\text{binomCdf}(n, p, 10, 20) \rightarrow 0.915292$

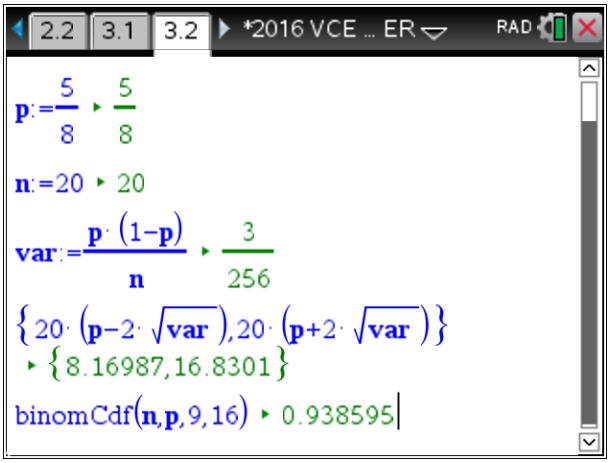
Part (a) (ii)

Working	**TI-Nspire CAS screenshot(s)**
$\Pr(X \geq 15 \mid X \geq 10)$ $= \frac{\Pr(X \geq 15)}{\Pr(X \geq 10)}$ $= 0.195$ (correct to 3 dp)	$\frac{\text{binomCdf}(n, p, 15, 20)}{\text{binomCdf}(n, p, 10, 20)} \rightarrow 0.19531$

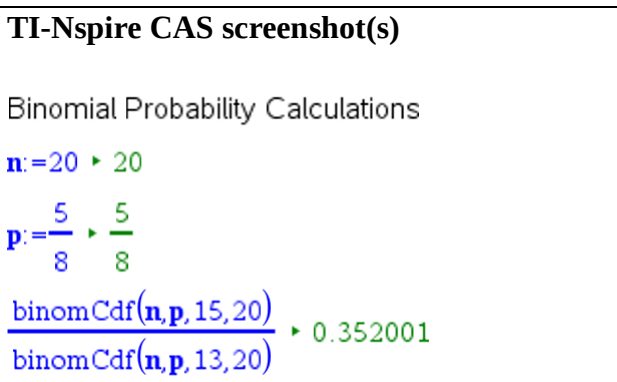
Part (a) (iii)

Working	**TI-Nspire CAS screenshot(s)**
$\text{E}(\hat{P}) = \frac{5}{8}$ $\text{var}(\hat{P}) = \frac{\frac{5}{8}\left(1 - \frac{5}{8}\right)}{20}$ $= \frac{3}{256}$	Sampling distribution calculations $p := \frac{5}{8} \rightarrow \frac{5}{8}$ $n := 20 \rightarrow 20$ $\text{var} := \frac{p \cdot (1-p)}{n} \rightarrow \frac{3}{256}$

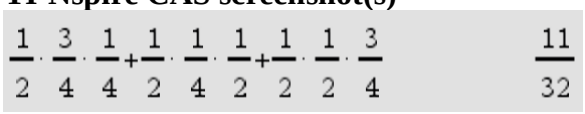
Part (a) (iv)

Working	**TI-Nspire CAS screenshot(s)**
Let X be the number from the sample that complete **S** in less than 3 mins. $\hat{P} = \frac{X}{20} \Rightarrow X = 20\hat{P}$ and $X \sim \text{Bi}\left(20, \frac{5}{8}\right)$ $\Pr\left(20\left(\frac{5}{8} - 2\sqrt{\frac{3}{256}}\right) \le X \le 20\left(\frac{5}{8} + 2\sqrt{\frac{3}{256}}\right)\right)$ $= \Pr(9 \le X \le 16)$ $= 0.939$ (correct to 3 dp)	

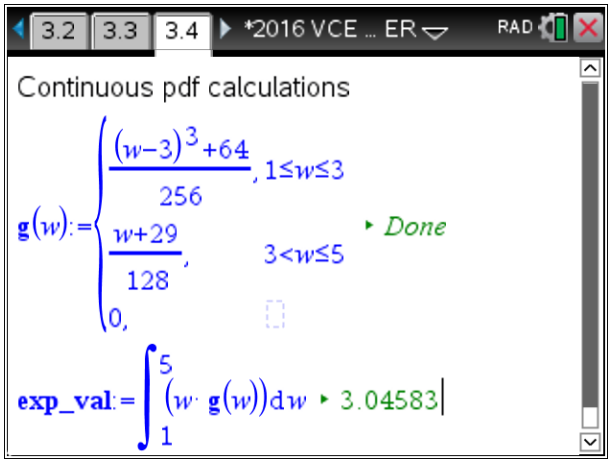
Part (a) (v)

Working	**TI-Nspire CAS screenshot(s)**
$\Pr\left(\hat{P} \ge \frac{3}{4} \mid \hat{P} \ge \frac{5}{8}\right)$ $= \Pr\left(X \ge 20 \times \frac{3}{4} \mid X \ge 15 \times \frac{3}{4}\right)$ $= \Pr(X \ge 15 \mid X \ge 13)$ $= 0.352$ (correct to 3 dp)	

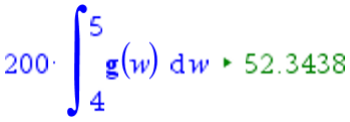
Part (b)

Working	**TI-Nspire CAS screenshot(s)**
$\frac{3}{32} + \frac{1}{16} + \frac{3}{16} = \frac{11}{32}$	

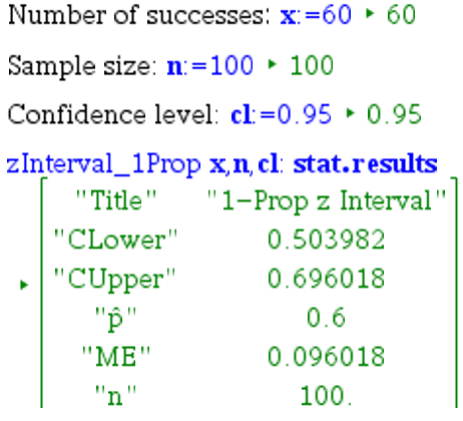
Part (c) (i)

Working	**TI-Nspire CAS screenshot(s)**
$\mathrm{E}(W)=\int_1^3 wg(w)\,dw+\int_3^5 wg(w)\,dw$ $=3.0458$ (correct to 4 dp) The Notes application can be used to solve questions involving continuous probability density functions.	

Part (c) (ii)

Working	**TI-Nspire CAS screenshot(s)**
$200\Pr(W>4)=52.3438$ So 52 members (nearest integer).	

Part (d)

Working	**TI-Nspire CAS screenshot(s)**
$0.504\le p\le 0.696$ (correct to 3dp) Author comment: This question part is worth one mark and so we can just write the answer down.	

Question 4:

Part (a) (i)

Working	**TI-Nspire CAS screenshot(s)**
$1-\frac{1}{e^2}$	$f(x):=e^x$ *Done* $\int_{-2}^{0} f(x)\,dx$ $1-e^{-2}$

Part (a) (ii)

Working	**TI-Nspire CAS screenshot(s)**
$1-\frac{1}{e^2}$	

Part (a) (iii)

Working	**TI-Nspire CAS screenshot(s)**
$e-\frac{1}{e^2}$	$\int_{0}^{1} f(x)\,dx+\int_{-2}^{0} f(x)\,dx$ $e-e^{-2}$

Part (b) (i)

Working	**TI-Nspire CAS screenshot(s)**
Solving $g(x)=k(x)$ for x gives $x=\frac{a\pm\sqrt{a^2-4}}{2}$. The Notes application can be used to solve equations (use of the zeros command is shown here).	$g(x):=\ln(x)$ ▸ *Done* $h(x):=-\ln(a-x)$ ▸ *Done* $xp:=\text{zeros}(g(x)-h(x),x)$ $\left\{\frac{-\left(\sqrt{a^2-4}-a\right)}{2}, -a\le\sqrt{a^2-4}\le a\right\}$ $\left\{\frac{\sqrt{a^2-4}+a}{2}, -a\le\sqrt{a^2-4}\le a\right\}$

Part (b) (ii)

Working	**TI-Nspire CAS screenshot(s)**
$a^2-4>0 \Rightarrow a>2\,(a>0)$	solve$(a^2-4>0, a)\,\vert\, a>0$ $\quad a>2$

Part (c)

Working	**TI-Nspire CAS screenshot(s)**
Solving $\dfrac{\dfrac{a-\sqrt{a^2-4}}{2}+\dfrac{a+\sqrt{a^2+4}}{2}}{2}=\sqrt{2}$ for a gives $a=2\sqrt{2}$.	solve$\left(\dfrac{\dfrac{-\left(\sqrt{a^2-4}-a\right)}{2}+\dfrac{\sqrt{a^2-4}+a}{2}}{2}=\sqrt{2}, a\right)$ ▸ $a=2\cdot\sqrt{2}$ ⚠

Question 5:

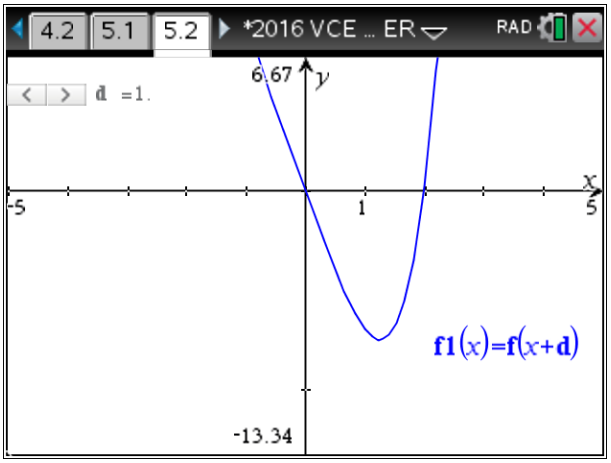
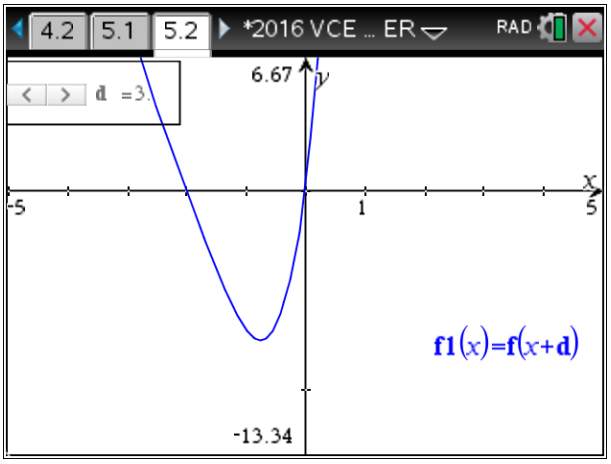
Part (a)

Working	**TI-Nspire CAS screenshot(s)**
$g(x)=x(x-2)\left((x+1)^2+3\right)$ The Notes application can be used to solve extended answer questions testing knowledge of calculus and algebra.	$f(x):=(x-3)\cdot(x-1)\cdot(x^2+3)$ ▸ *Done* $g(x):=x^4-8\cdot x$ ▸ *Done* factor$(g(x))$ ▸ $x\cdot(x-2)\cdot(x^2+2\cdot x+4)$ completeSquare$(x^2+2\cdot x+4, x)$ ▸ $(x+1)^2+3$

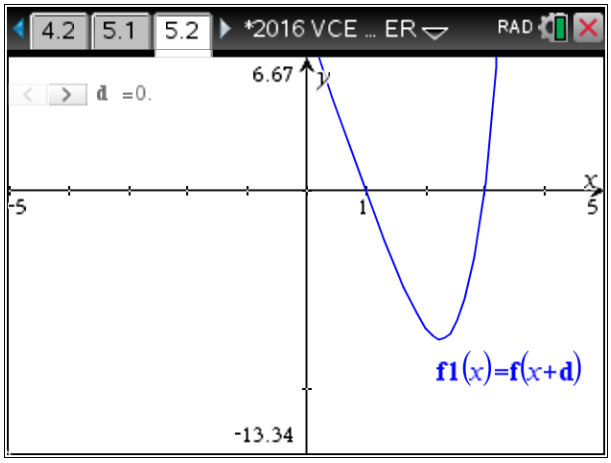
Part (b)

Working	**TI-Nspire CAS screenshot(s)**
Translation of 1 unit in the negative direction of the x-axis.	$f(x+1)=g(x)$ ▸ true

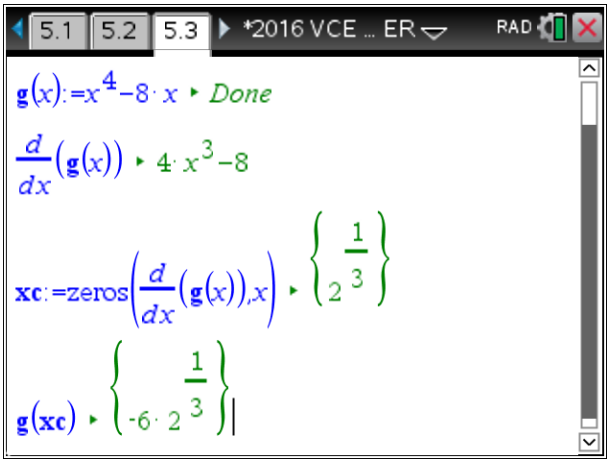
Part (c) (i)

Working	**TI-Nspire CAS screenshot(s)**
$1 \le d < 3$ If time permits, setting up a slider for the parameter d provides a good insight into the required values of d.	 

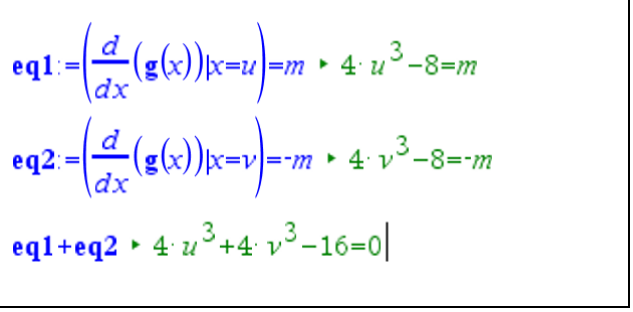
Part (c) (ii)

Working	**TI-Nspire CAS screenshot(s)**
$d < 1$	

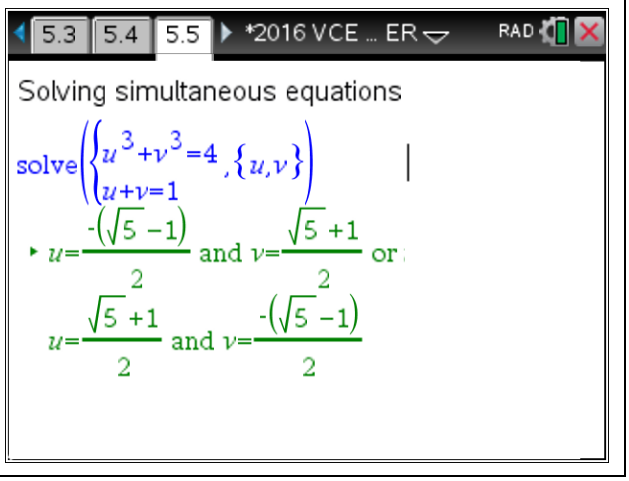
Part (d)

Working	**TI-Nspire CAS screenshot(s)**
Solving $g'(x)=0$ for x gives $x=2^{\frac{1}{3}}$. $n=g\left(2^{\frac{1}{3}}\right)=-6\left(2^{\frac{1}{3}}\right)$	

Part (e) (i)

Working	**TI-Nspire CAS screenshot(s)**
$g'(u)=m$ and $g'(v)=-m$ Adding the two equations and simplifying gives $u^3+v^3=4$.	

Part (e) (ii)

Working	**TI-Nspire CAS screenshot(s)**
Solving $u^3+v^3=4$ and $u+v=1$ for u and v with $m>0$ gives $u=\frac{\sqrt{5}+1}{2}$ and $v=\frac{-\sqrt{5}+1}{2}$.	

Part (f) (i)

Working	**TI-Nspire CAS screenshot(s)**
$y = g'(p)(x-p)+g(p)$ $= 4\left(p^3-2\right)x-3p^4$ The Notes application can be used to calculate equations of tangents and normals.	Equations of tangents and normals $\mathbf{f}(x):=x^4-8\cdot x \blacktriangleright$ *Done* $y=\text{tangentLine}(\mathbf{f}(x),x,p)$ $\blacktriangleright\ y=4\cdot\left(p^3-2\right)\cdot x-3\cdot p^4$

Part (f) (ii)

Working	**TI-Nspire CAS screenshot(s)**
Solving $4\left(p^3-2\right)\left(\frac{3}{2}\right)-3p^4=-12$ for p gives $p=0$ or $p=2$. When $p=0$, $y=-8x$. When $p=2$, $y=24x-48$.	5.4 | 5.5 | 5.6 | *2016 VCE ... ER | RAD $\mathbf{f}(x):=x^4-8\cdot x \blacktriangleright$ *Done* $y=\text{tangentLine}(\mathbf{f}(x),x,p)$ $\blacktriangleright\ y=4\cdot\left(p^3-2\right)\cdot x-3\cdot p^4$ $\mathbf{pval}:=\text{zeros}\left(y-\text{tangentLine}(\mathbf{f}(x),x,p),p\right)\mid x=\frac{3}{2}$ and $y=-12$ $\blacktriangleright\ \{0,2\}$ $y=\text{tangentLine}(\mathbf{f}(x),x,p)\mid p=\mathbf{pval}$ $\blacktriangleright\ y=\{-8\cdot x,24\cdot x-48\}$